wave, the maximum in the radiation pattern rises sharply as Ω is increased beyond Ω_3.

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Attenuation Constant of Lunar Line and *T*-Septate Lunar Line*

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Summary—The attenuation constant α of the lunar line and that of the *T*-septate lunar line were derived from the average power loss W_L and the average power transfer W_T in each line, that is the ratio, $W_L/2W_T$. The average power loss and the average power transfer for the lunar line and for the *T*-septate lunar line were derived from their respective field functions. The theoretical attenuation constant of a typical lunar line is less than 0.7 db/100 ft for frequencies greater than 2000 Mc. The theoretical attenuation constant of a typical *T*-septate line is less than 0.9 db/100 ft for frequencies greater than 1000 Mc. Experimental measurements of the attenuation constant of a *T*-septate lunar line agree with the theoretical value. In the 200 to 2000 Mc frequency band, the lunar line and the *T*-septate lunar line offer a compact and light package without an appreciable sacrifice in peak power handling capacity or attenuation.

INTRODUCTION

TWO NEW microwave transmission lines, lunar line and *T*-septate lunar line were developed at The Boeing Company. The lunar line is an eccentric version of Schelkunoff's coaxial cylinders with a radial baffle[1] and is formed by two eccentric circular metal tubes, which are either connected with a metal bar or tangential to each other. The *T*-septate or lazy-*T* lunar line is a modification of the lunar line and is also formed by two eccentric circular metal tubes. In the *T*-septate lunar line, however, part of the inner tube is cut out and a vertical metal bar is passed through the cut out section to connect the inner surfaces of the two tubes. The outer tube and the bar are made of brass, but the inner tube is made of copper which maintains the cylindric form after being cut. A perturbation method is used to obtain the dominant cutoff wavelength and the field functions of these lines.[2,3]

The main effect of the finite conductivity in the waveguide will be attenuation caused by the power loss in the conducting boundaries. The value of this attenuation may be estimated by the ratio of power loss per unit length to the average power transferred. For a good conductor, it is reasonable to assume that the expression for power transfer derived for the ideal guide applies well enough to the actual guide, and that power loss may be computed by taking the current flow of the ideal guide as flowing in the walls of the actual guide with a known conductivity.

In this paper, the attenuation constant α of the lunar line and of the *T*-septate lunar line are derived from the average power loss per unit length W_L and the average power transfer for the lunar line and for the *T*-septate lunar line with their respective field functions. The numerically calculated dissipative attenuation constants of the *T*-septate lunar line for different frequencies are consistent with experimental results. Com-

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[2] A. Y. Hu and A. Ishimaru, "The Dominant Cut-Off Wavelength of a Lunar Line," The Boeing Co., Seattle, Wash., Tech. Rept. No. 1, Contract No. AF 19(604)-6189, AFCRL 133, Boeing Doc. D6-7467; published in IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 552-556; November, 1961.
[3] A. Y. Hu and D. E. Isbell, "The Dominant Cut-Off Wavelength of a *T*-Septate Lunar Line," The Boeing Co., Seattle, Wash., Tech. Rept. No. 3, Contract No. AF 19(604)-6189, AFCRL-62-80, Boeing Doc. D6-8188; January, 1962.

pared with the equivalent rectangular waveguide at the same operating frequency, the attenuation constant of lunar line and T-septate lunar line is increased, and the power-handling capability is decreased, but cross section and weight of both lines are reduced. Compared with the equivalent rectangular ridge waveguide at the same frequency, the attenuation constant and the size may be the same, but lunar line and T-septate lunar line have a higher power-handling capability and can be pressurized without deformation. From a practical consideration, lunar line and T-septate lunar line are likely to have their greatest application in 200–2000 Mc frequency band. This band is also of rapidly expanding interest for space radar and space communications.

Theoretical Analysis

Dominant Transverse Electrical Wave

Assume only the dominant TE mode is propagating in the lunar line and the T-septate lunar line, and that the guide wall is perfectly conducting. The z component of the magnetic field H_z perpendicular to the cross section of the line, satisfies the two-dimensional wave equation:

$$(\nabla_t^2 + \beta_c^2)H_z = 0; \qquad \frac{\partial H_z}{\partial n} = 0 \tag{1}$$

where Δ_t is the gradient operator transverse to the z axis, β_c is the cutoff wave number, and the cutoff wavelength is given by $\lambda_c = (2\pi/\beta_c)$. An approximate solution is obtained by deforming the boundary of the cross section into a series of steps (Figs. 1 and 2). The lunar line is deformed by eighteen fan-shaped regions, and the T-septate lunar line is deformed by twenty-two regions. Because of their symmetry about $\phi = \pi$, only half of the regions must be considered. For the cylindrical coordinate system, the ith region is defined by:

$$\phi_{i-1} \leq \phi \leq \phi_i$$
$$\rho_0 \leq \rho \leq \rho_i$$

where

$$i = 1, \cdots, 9 \text{ for lunar line}$$

and

$$i = 1, \cdots, 11 \text{ for } T\text{-septate lunar line.}$$

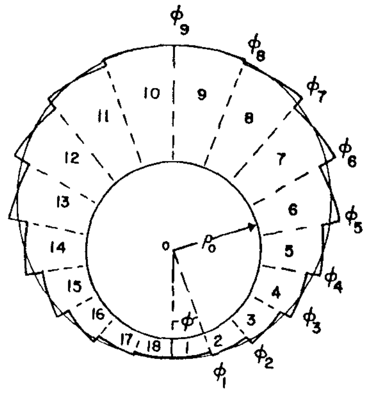
The solution of (1) for each region is:[4]

$$H_{z_i} = \sum_{n=1}^{\infty} C_{in} Z_{p_{in}}(\beta_c \rho) \cos p_{in}(\phi - \alpha_{in}) \tag{2}$$

where p is the order of the Bessel function and C is the coefficient of the series solution. The function Z_p represents the following combination of Bessel functions:

$$Z_{p_{in}}(\beta_c\rho) = J'_{p_{in}}(\beta_c\rho_0) N_{p_{in}}(\beta_c\rho) - N'_{p_{in}}(\beta_c\rho_0) J_{p_{in}}(\beta_c\rho). \tag{3}$$

[4] N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Company, New York, N. Y., pp. 66–80; 1951.



$\rho_0 = 0.8125''$.
ϕ_i = length from center O to outside of ith fan-shape region.

Fig. 1—Cross section of an eighteen-stepped form of a lunar line.

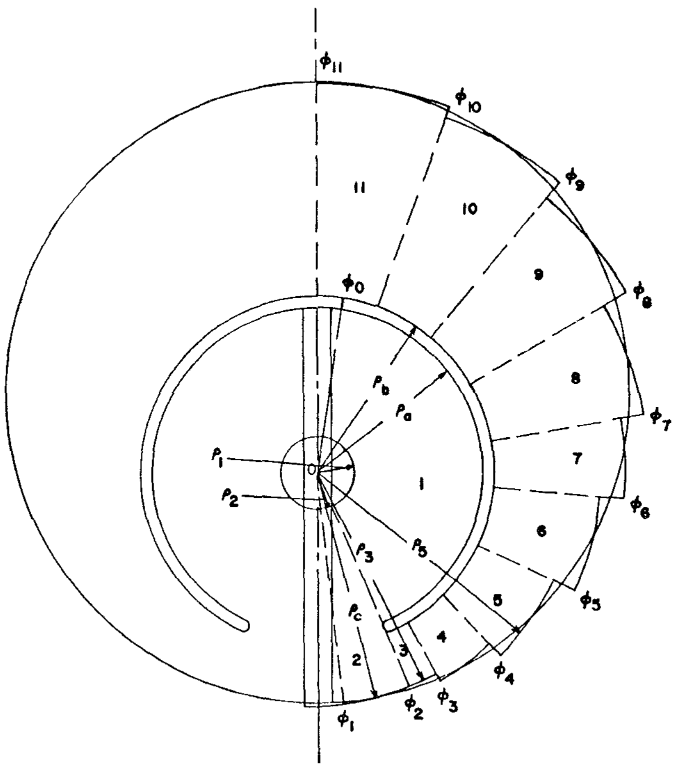


Fig. 2—Cross section of an eleven stepped fan-shaped form of the half cross section of a T-septate lunar line.

The cutoff wave number β_c the order of Bessel function p, and the angular parameter α_i for each region have been determined[2,3] and are listed in Tables I and II. The transverse fields of the wave can be written as:

$$E_{\rho_i} = -\frac{1}{\beta_c^2}\frac{j\omega\mu}{\rho}\frac{\partial H_{z_i}}{\partial \phi} \tag{4}$$

$$E_{\phi_i} = -\frac{j\omega\mu}{\beta_c^2}\frac{\partial H_{z_i}}{\partial \rho} \tag{5}$$

$$H_{\rho_i} = -\frac{\gamma}{\beta_c^2}\frac{\partial H_{z_i}}{\partial \rho} \tag{6}$$

$$H_{\phi_i} = -\frac{\gamma}{\beta_c^2 \rho}\frac{\partial H_{z_i}}{\partial \phi} \tag{7}$$

TABLE I

Parameters of a Lunar Line

$\beta_c = 0.67$

i	ρ_i, in.	$\beta_c\rho_i$	P_{i1}, in.	α_i	ϕ_i	ϕ_i^*	C_0
0	0.8125	0.5444					
1	0.9800	0.6566	0.599	0.0000	20°	20.06°	1.0000 C_1
2	1.0400	0.6968	0.621	0.1087	40°	39.87°	1.0029 C_1
3	1.1200	0.7504	0.642	0.1735	60°	60.21°	0.8923 C_1
4	1.2300	0.8241	0.677	0.4787	80°	80.66°	0.8703 C_1
5	1.3600	0.9112	0.718	0.7534	100°	101.10°	0.8227 C_1
6	1.5300	1.0251	0.771	1.0161	120°	120.81°	0.7446 C_1
7	1.6800	1.1256	0.814	1.1657	140°	140.70°	0.6694 C_1
8	1.8100	1.2127	0.851	1.2935	160°	160.30°	0.5996 C_1
9	1.8600	1.2462	0.863	1.3363	180°	180.29°	0.5690 C_1

TABLE II

Parameters of a T-Septate Lunar Line

$\beta_c = 0.40$ $\eta = -6.889$

i	ρ_i, in.	$\beta_c\rho_i$	P_{i1}, in.	α_i	ϕ_i	ϕ_i^*	C_i
a	0.7600	0.304			$\phi_0 = 172°$	$\phi_0^* = 176°$	
1	0.1600	0.064	0.16875	+3.00197	6°	3°	0.6105 C_3
c	1.0800	0.432					
2	0.1600	0.064	0.22020	+0.10472	22.5°		0.8059 C_3
b	0.8125	0.325					
3	1.0800	0.432	0.37720	−2.02241	30°	30.17°	1.0000 C_3
4	1.1200	0.448	0.38480	−1.79778	45°	45.25°	0.9172 C_3
5	1.2000	0.480	0.40000	−1.41853	65°	65.31°	0.7864 C_3
6	1.3100	0.524	0.42050	−1.01794	85°	85.42°	0.6620 C_3
7	1.4200	0.568	0.44080	−0.69190	100°	100.34°	0.5717 C_3
8	1.5300	0.612	0.46100	−0.41621	120°	120.45°	0.5026 C_3
9	1.6500	0.660	0.48285	−0.17202	140°	140.32°	0.4403 C_3
10	1.7500	0.700	0.50090	−0.00728	160°	160.06°	0.3959 C_3
11	1.8000	0.720	0.50985	+0.06069	180°	179.99°	0.3748 C_3

where γ is the propagation constant equal to $\sqrt{\beta_c^2 - K^2}$ and $K = \omega\sqrt{\mu\epsilon}$. Eqs. (6) and (7) may be written in the vector form,

$$\overline{H}_{t_i} = -\frac{\gamma}{\beta_c^2}\nabla_t H_{z_i} \tag{8}$$

where $\overline{H}_t$ is the transverse magnetic field to the z axis.

Power Transfer in TE Waves[5]

The average power transfer for each region in the propagating range is, as usual, obtained from the Poynting vector:

$$\begin{aligned} W_{T_i} &= \frac{1}{2}\int_{\text{c.s.}} R_e(\overline{E} \times \overline{H})\cdot\overline{ds} \\ &= \frac{1}{2} Z_{TE}\int_{\text{c.s.}} |\overline{H}_t|^2 ds. \end{aligned} \tag{9}$$

Using the divergence theorem, (9) becomes

$$\begin{aligned} W_{T_i} = \frac{1}{2} Z_{TE}\frac{\gamma^2}{\beta_c^4} &\cdot \left\{\beta_c^2\int_{\text{c.s.}} H_{z_i}^2 ds + \int_{\rho_0}^{\rho_{i-1}} H_{z_i}\frac{\partial H_{z_i}}{\rho\partial\phi}d\rho\Big|_{\phi=\phi_{i-1}} \right. \\ &\left. - \int_{\rho_0}^{\rho_i} H_{z_i}\frac{\partial H_{z_i}}{\rho\partial\phi}d\rho\Big|_{\phi=\phi_i}\right\}. \end{aligned} \tag{10}$$

[5] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, N. Y., pp. 47, 240, 344–354, and 358; 1953.

Between the common boundary $\phi = \phi_i$, the integral

$$\int H_z\frac{\partial H_z}{\rho\partial\phi}d\rho$$

for the ith and the $(i+1)$th region cancels out. Therefore, the average power transfer for the lunar line is:

$$W_{TL} = \sum_{i=1}^{18} W_{T_i} = Z_{TE}\frac{\gamma^2}{\beta_c^2}\sum_{i=1}^{9}\int_{\text{c.s.}} H_{z_i}^2 ds. \tag{11}$$

The average power transfer for the T-septate lunar line is:

$$W_{TT} = \sum_{i=1}^{22} W_{T_i} = Z_{TE}\frac{\gamma^2}{\beta_c^2}\sum_{i=1}^{11}\int_{\text{c.s.}} H_{z_i}^2 ds. \tag{12}$$

Power Loss Per Unit Length in TE Waves

When the conducting boundaries are imperfect, and an exact solution requires the use of Maxwell's equations in both the dielectric and conducting domains of the waveguide, it has not been found possible to solve for the general case. Fortunately, most practical conductors are good enough to cause only a slight modification of the ideal solution and the approximated formula for the dissipative attenuation constant α can be used:

$$\alpha = \frac{W_L}{2W_T}\cdot \tag{13}$$

To compute the average power loss per unit length, the current flow in the guide walls must be known; it is assumed to be the same as that in the ideal guide. By the $\bar{n}\times\bar{H}$ rule, there is an axial current arising from the transverse component of the magnetic field and a transverse current arising from the axial magnetic field at the boundary.

Hence, the power loss per unit length for the ith region is,

$$W_{L_i} = \frac{1}{2}\Bigg\{R_{sb}\int_{\phi_{i-1}}^{\phi_i} H_{z_i}{}^2\rho d\phi\Big|_{\rho=\rho_i} + R_{sc}\int_{\phi_{i-1}}^{\phi_i} H_{z_i}{}^2\rho d\phi\Big|_{\rho=\rho_0} + R_{sb}\int_{\rho_{i-1}}^{\rho_i} H_{z_i}{}^2 d\rho\Big|_{\phi=\phi_{i-}} + \left(\frac{f}{f_c}\right)^2 \frac{\left[1-\left(\frac{f_c}{f}\right)^2\right]}{\beta_c{}^2} \cdot\Bigg[\int_{\phi_{i-1}}^{\phi_i} R_{sb}\left(\frac{\partial H_{z_i}}{\rho\partial\phi}\right)^2 \rho d\phi\Big|_{\rho=\rho} + \int_{\phi_{i-1}}^{\phi_i} R_{sc}\left(\frac{\partial H_{z_i}}{\rho\partial\phi}\right)^2 \rho d\phi\Big|_{\rho=\rho_0} + \int_{\rho_{i-1}}^{\rho_i} R_{sb}\left(\frac{\partial H_{z_i}}{\partial\rho}\right)^2 d\rho\Big|_{\phi=\phi_{i-1}}\Bigg]\Bigg\} \quad (14)$$

where R_{sb} is the surface resistivity of brass and R_{sc} is the surface resistivity of copper.

The power loss per unit length for the lunar line is:

$$W_{LL} = 2\sum_{i=1}^{9} W_{L_i}. \quad (15)$$

The power loss per unit length for the T-septate lunar line is:

$$W_{LT} = 2\sum_{i=1}^{11} W_{L_i}. \quad (16)$$

Attenuation Constant

By substituting the power loss and the power transfer into (13), the attenuation constant α can be obtained. Both the denominator and the numerator of the attenuation constant are a series summation of the ith region. The coefficient C of different regions must be in terms of the coefficient of one region. Using the orthogonality of the function Z_{p_i}, C_{i+1} can be written in terms of C_i.[2,3] Thus:

$$C_{i+1} = C_i \frac{\sin p_i(\phi_i - \alpha_i)}{\sin p_{i+1}(\phi_i - \alpha_{i+1})} \frac{2p_i}{[I_{p_{i+1}}(\rho_{i+1}) - I_{p_{i+1}}(\rho_0)]} \cdot \frac{\beta_c\rho_i Z_{p_i}(\beta_c\rho_i) Z'_{p_{i+1}}(\beta_c\rho_i)}{[(p_{i+1})^2 - (p_i)^2]} \quad (17)$$

where $\rho_{i+1} > \rho_i$, for both lines,

$$C_1 = \frac{C_3 p_3 \sin p_3(\phi_2 - \alpha_3)}{\eta_{p_1} \sin p_1(\phi_2 - \alpha_1)}, \text{ for the } T\text{-septate lunar line} \quad (18)$$

and:

$$C_2 = \frac{2C_3 p_3 \sin p_3(\phi_2 - \alpha_3)}{[\sin p_2(\phi_2 - \alpha_2)][I_{p_2}(\rho_c) - I_{p_2}(\rho_2)]} \cdot \left\{\frac{\beta_c\rho_b Z_{p_3}(\beta_c\rho_b) Z_{p_2}'(\beta_c\rho_b)}{(p_3)^2 - (p_2)^2} - \frac{\beta_c\rho_a Z_{p_1}(\beta_c\rho_a) Z_{p_2}'(\beta_c\rho_a)}{\eta[(p_1)^2 - (p_2)^2]}\right\}$$

for the T-septate lunar line. (19)

The $I_{p_i}(\rho)$ function represents an integral,[2,3,6] thus:

$$\int_{\rho_0}^{\rho_i} Z_{p_i}{}^2(\beta_c\rho)\frac{d\rho}{\rho} = \frac{1}{2p_i}[I_{p_i}(\rho_i) - I_{p_i}(\rho_0)]. \quad (20)$$

The coefficients of the ith region of the lunar line can be written in terms of C_1 and the coefficients of the ith region of the T-septate lunar line can be written in terms of C_3. Therefore, the coefficient C_1 or C_3 of the summation of the denominator and numerator of the attenuation constant can be factored out and cancel each other. Then, (11) for the lunear line power transfer becomes:

$$W_{TL} = C_1{}^2\eta\left(\frac{f}{f_c}\right)^2\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sum_{i=1}^{9}\left\{\left\{\prod_{j=1}^{i-1}\left\{\frac{\sin p_j(\phi_j - \alpha_j)}{\sin p_{j+1}(\phi_j - \alpha_{j+1})} \frac{2p_j\beta_c\rho_j Z_{p_j}(\beta_c\rho_j Z'_{p_{j+1}}(\beta_c\rho_j)}{[I_{p_{j+1}}(\rho_{j+1}) - I_{p_{j+1}}(\rho_0)][(p_j)^2 - (p_{j+1})^2]}\right\}^2\right\} \cdot \int_{\rho_0}^{\rho_i}\int_{\phi_{i-}}^{\phi_i} Z_{p_i}{}^2(\beta_c\rho)\cos^2 p_i(\phi - \alpha_i)\rho d\rho d\phi\right\}. \quad (21)$$

Similarly, (12) for the T-septate lunar line power transfer becomes:

[6] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, Mass.; 1958.

$$
\begin{aligned}
W_{TT} = \eta\left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(\frac{f_c}{f}\right)^2}\Bigg\{ & C_1^2 \int_{\rho_1}^{\rho_a}\int_{\phi_1}^{\phi_0} Z_{p_1}^2(\beta_c\rho)\cos^2 p_1(\phi-\alpha_1)\rho d\rho d\phi \\
& + C_2^2 \int_{\rho_2}^{\rho_c}\int_{\phi_1}^{\phi_2} Z_{p_2}^2(\beta_c\rho)\cos^2 p_2(\phi-\alpha_2)\rho d\rho d\phi \\
& + C_3^2 \left\{ \prod_{i=3}^{11} \left\{ \prod_{j=3}^{i-3} \left\{ \frac{\sin p_j(\phi_j-\alpha_j)}{\sin p_{j+1}(\phi_j-\alpha_{j+1})} \frac{2p_j\beta_c\rho_j Z_{p_j}(\beta_c\rho_j)Z'_{p_j+1}(\beta_c\rho_j)}{[I_{p_j+1}(\rho_{j+1}) - I_{p_j+1}(\rho_b)][(p_j)^2-(p_{j+1})^2]} \right\}^2 \right\} \right. \\
& \left. \cdot \int_{\rho_b}^{\rho_i}\int_{\phi_{i-1}}^{\phi_i} Z_{p_i}^2(\beta_c\rho)\cos^2 p_i(\phi-\alpha_i)\rho d\rho d\phi \right\}\Bigg\}.
\end{aligned} \tag{22}
$$

The power loss in the ith region is evaluated along the paths of line AB, arc BC, and arc GH as shown in Fig. 3. The actual path is arc DE and GH. In order to simplify the computation, arc BC is extended to F so that arc BF equals arc DE, that is:

$$
\rho_i(\phi_i^* - \phi_{i-1}^*) = \rho_0'(\phi_i' - \phi_{i-1}'). \tag{23}
$$

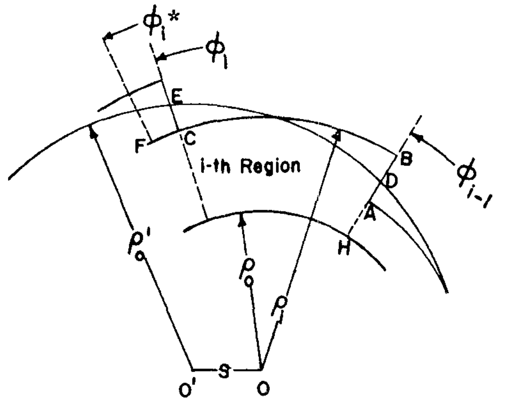


Fig. 3—The ith region of a lunar line or of a T-septate lunar line.

The angle P_i' is calculated from the angle ϕ_i when the origin point o transfers to the origin point o'.

Then, (15) for lunar line power loss will be reduced to:

$$
\begin{aligned}
W_{LL} = C_1^2 \sum_{i=1}^{9} \Bigg\{ & \left\{ \prod_{j=1}^{i-1} \left\{ \frac{\sin p_j(\phi_j-\alpha_j)}{\sin p_{j+1}(\phi_j-\alpha_{j+1})} \frac{2p_j\beta_c\rho_j Z_{p_j}(\beta_c\rho_j)Z'_{p_j+1}(\beta_c\rho_j)}{[I_{p_j+1}(\rho_{j+1}) - I_{p_j+1}(\rho_0)][(p_j)^2-(p_{j+1})^2]} \right\}^2 \right\} \\
& \cdot \Bigg\{ [R_{sb}\rho_i Z_{p_i}^2(\beta_c\rho_i) + R_{sc}\rho_0 Z_{p_i}^2(\beta_c\rho_0)] \int_{\phi_{i-1}}^{\phi_i^*} \cos^2 p_i(\phi-\alpha_i)d\phi + \delta_{i_1} R_{sb}\cos^2 p_1(\phi_0-\alpha_1)\int_{\rho_0}^{\rho_1} Z_{p_1}^2(\beta_c\rho)d\rho \\
& + \left(\frac{f}{f_c}\right)^2 \frac{\left[1-\left(\frac{f_c}{f}\right)^2\right]}{\beta_c^2} \Bigg\{ p_i^2 \left[\frac{1}{\rho_i} Z_{p_i}^2(\beta_c\rho_i) + \frac{1}{\rho_0} Z_{p_i}^2(\beta_c\rho_0)\right]\left[\int_{\phi_{i-1}}^{\phi_i^*} \sin^2 p_i(\phi-\alpha_i)d\phi \right. \\
& \left. + \delta_{i_1} R_{sb}\beta_c^2 \cos^2 p_1(\phi_0-\alpha_1)\int_{\rho_0}^{\rho_1} Z_{p_1}'^2(\beta_c\rho)d\rho\right]\Bigg\}\Bigg\}\Bigg\},
\end{aligned} \tag{24}
$$

where:

$$
\delta_{i_1} = \begin{cases} 1 & \text{for } i = 1 \\ 0 & \text{for } i \neq 1. \end{cases} \tag{25}
$$

Similarly, the bar of regions 1 and 2 in the T-septate lunar line is extended to its actual length, and (16) for the line power loss is reduced to:

$$
\begin{aligned}
W_{LT} = C_1^2 \Bigg\{ \bigg[& R_{sc}\rho_a Z_{p_1}^2(\beta_c\rho_a) \int_{\phi_2}^{\phi_0^*} \cos^2 p_1(\phi-\alpha_1)d\phi + R_{sb}\cos^2 p_1(\phi_0-\alpha_1)\int_{-\rho_1}^{\rho_a} Z_{p_1}^2(\beta_c\rho)d\rho \bigg] + \left(\frac{f}{f_c}\right)^2 \frac{\left[1-\left(\frac{f_c}{f}\right)^2\right]}{\beta_c^2} \\
& \cdot \bigg[p_1^2 \frac{R_{sc}}{\rho_a} Z_{p_1}^2(\beta_c\rho_a) \int_{\phi_2}^{\phi_0^*} \sin^2 p_1(\phi-\alpha_1)d\phi + R_{sb}\beta_c^2\cos^2 p_1(\phi_0-\alpha_1)\int_{\rho_1}^{\rho_a} Z_{p_1}'^2(\beta_c\rho)d\rho \bigg] \Bigg\} \\
& + C_2^2 R_{s_b} \bigg\{ \rho_c Z_{p_2}^2(\beta_c\rho_c)\int_{\phi_1^*}^{\phi_2} \cos^2 p_2(\phi-\alpha_2)d\phi + \cos^2 p_2(\phi_1-\alpha_2)\int_{\rho_2}^{\rho_c} Z_{p_2}^2(\beta_c\rho)d\rho + \left(\frac{f}{f_c}\right)^2 \frac{\left[1-\left(\frac{f_c}{f}\right)^2\right]}{\beta_c^2} \\
& \cdot \bigg[\frac{p_2^2}{\rho_c} Z_{p_2}^2(\beta_c\rho_c)\int_{\phi_1^*}^{\phi_2}\sin^2 p_2(\phi-\alpha_2)d\phi + \beta_c^2\cos^2 p_2(\phi_1-\alpha_2)\int_{\rho_2}^{\rho_c} Z_{p_2}'(\beta_c\rho)d\rho \bigg] \bigg\} \\
& + C_3^2 \sum_{i=3}^{11} \Bigg\{ \Bigg\{ \prod_{j=3}^{i-3} \bigg\{ \frac{\sin p_j(\phi_j-\alpha_j)}{\sin p_{j+1}(\phi_j-\alpha_{j+1})} \; \frac{2p_j\beta_c\rho_j Z_{p_j}(\beta_c\rho_j)Z'_{p_{j+1}}(\beta_c\rho_j)}{[I_{p_{j+1}}(\rho_{j+1}) - I_{p_{j+1}}(\rho_b)][(p_j)^2-(p_{j+1})^2]} \bigg\}^2 \Bigg\} \\
& \cdot \bigg\{ [R_{sb}\rho_i Z_{p_i}^2(\beta_c\rho_i) + R_{sc}\rho_b Z_{p_i}^2(\beta_c\rho_b)] \int_{\phi_{i-1}}^{\phi_i^*} \cos^2 p_i(\phi-\alpha_i)d\phi + \left(\frac{f}{f_c}\right)^2 \frac{\left[1-\left(\frac{f_c}{f}\right)^2\right] p_i^2}{\beta_c^2} \\
& \cdot \bigg\{ \bigg[\frac{R_{sb}}{\rho_i} Z_{p_i}^2(\beta_c\rho_i) + \frac{R_{sc}}{\rho_b} Z_{p_i}^2(\beta_c\rho_b) \bigg] \int_{\phi_{i-1}}^{\phi_i^*} \sin^2 p_i(\phi-\alpha_i)d\phi \bigg\} \Bigg\} . \qquad (26)
\end{aligned}
$$

The attenuation constant for the lunar line is obtained by substituting (21) and (24) into (13). Similarly, the attenuation constant for the T-septate lunar line can be obtained by substituting (22) and (26) into (13).

Sample Computation

The angle ϕ_i^* and the coefficients C_i of both lines are computed and listed in Tables I and II. Let,

$$A = \sum_i \int_{c.s.} H_{z_i}^2 ds, \qquad (27)$$

$$B = \sum_i \oint R_s H_{z_i}^2 dl, \qquad (28)$$

$$C = \sum_i \oint R_s \left[\frac{\partial H_{z_i}}{\partial l} \right] dl. \qquad (29)$$

The values A, B and C are computed by using[6] data found in Tables I and II; the surface resistivity[5] of brass is $5.01\times10^{-7}\sqrt{f}$, and the surface resistivity of copper is $2.6\times10^{-7}\sqrt{f}$. The results are listed in Table III.

The attenuation constants of the lunar line and of the T-septate lunar line can be simplified to:

$$\alpha = \frac{\left\{\left(\frac{f_c}{f}\right)^2 B + \left[1-\left(\frac{f_c}{f}\right)^2\right]\frac{C}{\beta_c^2}\right\}\sqrt{f}}{2(377)A\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \times 8.686 \times 12 \times 10^{-7} \text{ db/ft}. \qquad (30)$$

TABLE III

A, *B*, *C*, PARAMETERS OF THE LUNAR LINE AND OF THE *T*-SEPTATE LUNAR LINE

	Lunar Line	*T*-Septate Lunar Line
A	1.24	2.22
B	17.65	35.84
C	3.42	4.84

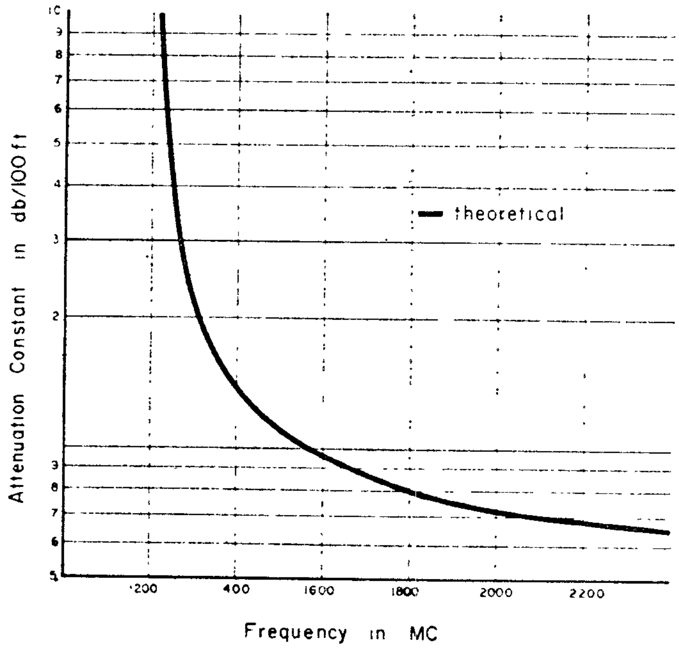


Fig. 4—Attenuation of lunar line vs frequency.

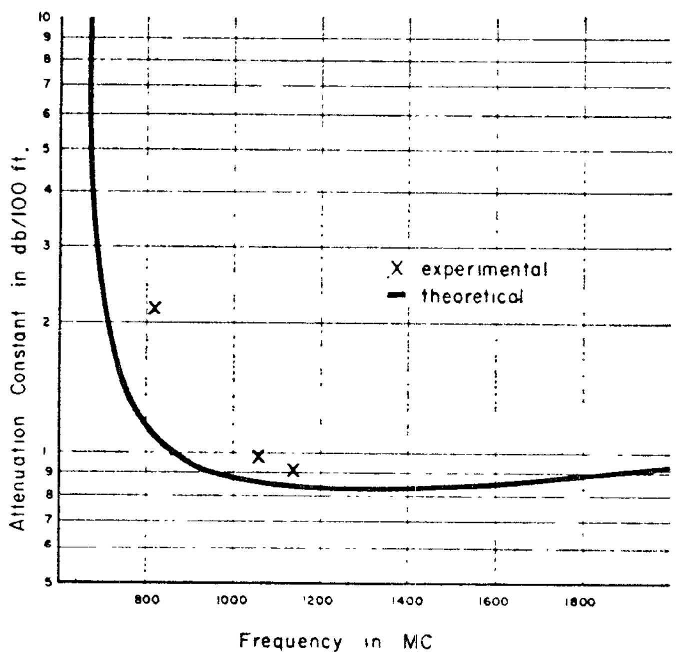


Fig. 5—Attenuation of *T*-septate lunar line vs frequency.

The attenuation constants are computed for different frequencies and the theoretical curves are plotted in Figs. 4 and 5.

EXPERIMENTAL MEASUREMENT OF THE *T*-SEPTATE LUNAR LINE ATTENUATION CONSTANT

The power-ratio method[7,8] was used to measure the attenuation constant of a *T*-septate lunar line at several different frequencies (Fig. 6). Both ends of *T*-septate lunar lines *A* and *B* in Fig. 6 were terminated in a short circuit, and the length L between ends was adjusted to be a multiple of a half of the waveguide wavelength. Preliminary attenuation-constant measurements were made exciting the field in the guide with one probe and measuring the power with another. However, mechanical difficulties associated with changing the position of the probes in the waveguide made measurements with adjustable-loops more convenient. The length l of the loop transition between the coaxial cable and the *T*-septate lunar line is approximately one quarter of the waveguide wavelength so that the maximum magnetic flux is linked from, or into, the lunar line; the leads from the loop are taken from the short-circuit end. Because the input impedance of the loop transition was not the desired 50 ohms, a double-stub tuner was used to achieve an impedance match and the maximum power transfer. The double-stub tuner and the loop may be considered as a combined transition unit.

The input power at *T*-septate lunar lines *A* and *B* was kept equal and constant during tests at the same frequency. The length l of the loop transition was set at 3 inches, and the input frequency was varied to get the maximum output power. The optimum frequency is 1140 Mc and is measured at a waveguide wavelength of 12.8 inches. The length L_1 of *T*-septate lunar line cavity *A* was then readjusted for the maximum output power. The output power reading P_1 was taken when the double-stub tuner had been adjusted to obtain the lowest VSWR below 1.06 for the input and output sides. The output power reading P_2 was taken after carefully switching the test set-up from line *A* to line *B* and changing the length L_2 of cavity *B* for the maximum output power. The cable connecting the tuner and the loop must be the same length for test line *A* and test line *B*. Therefore two *T*-septate lunar lines must be used to avoid changing the cable length.

The attenuation constant α is a measure of the dissipative attenuation per unit length of the line, and is given by:

$$\alpha = [10 \log_{10} (P_2/P_1)]/\Delta L \text{ db/ft} \tag{31}$$

where ΔL is the difference between L_1 and L_2.

[7] C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., New York, N. Y., pp. 680–805; 1947.

[8] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., New York, N. Y., pp. 467–470; 1957.

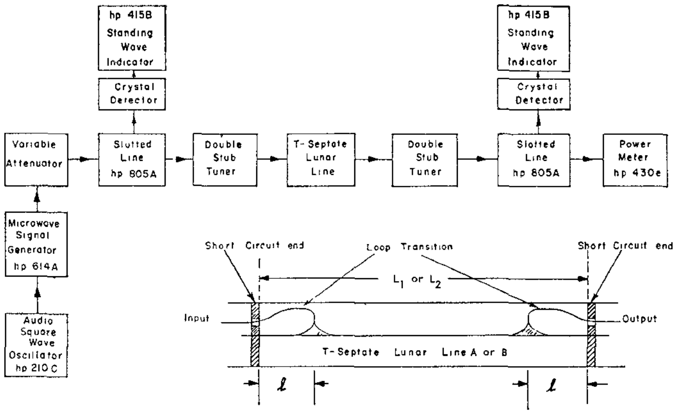


Fig. 6—Block diagram of a method to measure the attenuation constant of a T-septate lunar line.

Loop transistions of three different lengths were tested. The attenuation constants obtained from direct measurements of the power ratios of the two T-septate lunar lines are plotted in Fig. 5. The values calculated from direct measurements agree with those obtained theoretically.

Discussion and Conclusion

The attenuation constant, size, weight and power-handling capability of lunar line and T-septate lunar line are compared with a rectangular waveguide and a ridge waveguide as follows: the theoretical attenuation constant of the lunar line made of copper and brass is about 0.7 db/100 ft at 2000 Mc. If the lunar line is made completely of copper, the attenuation constant will be approximately 0.4 to 0.45 db/100 ft which is the same as the theoretical attenuation constant of Microwave Development Laboratories rectangular waveguide Type WR-430. However, the cross section and width of the lunar line are only about 1/1.5th that of the WR-430 waveguide.

The attenuation constant of Airtron Inc.[9] aluminum double ridge waveguide ARA-1109 at 2000 Mc is 0.97 db/100 ft, and its peak power handling capability is 14,000 watts. The attenuation constant of Airtron, Inc., aluminum single ridge waveguide 50375 at 2000 Mc is 5 db/100 ft and its peak power handling capability is 1500 watts. The attenuation constant of lunar line is about the same as ARA-1109 double ridge waveguide and its power handling capability is greater than ARA-1109 waveguide. Both attenuation constant and power handling capability are better than 50375 waveguide.

The theoretical attenuation constant of the copper and brass T-septate lunar line is about 0.9 db/100 ft at 1000 Mc. If the T-septate lunar line is made completely of copper, the attenuation constant will be approximately 0.5 to 0.6 db/100 ft. The attenuation constant of Andrew rectangular waveguide Type 975 which is made of copper clad steel is 0.11 db/100 ft. Therefore, although the attenuation constant of the T-septate lunar line is about 5 times that of Type 975 waveguide, its cross section is only about $\frac{1}{7}$th that of the 975 waveguide.

The power handling capability of six-inch outside diameter T-septate lunar line, which has twice the cross section of the line in this analysis, was tested. The peak power was about 4 megawatts. T-septate lunar line broke down at the center outside surface of the inner conductor.

For aircraft application, in order to increase the power handling capability of waveguide components of radar systems, pressurization must be introduced. Unfortunately, rectangular waveguide is almost the poorest possible shape for pressure vessel design. But, lunar line can be pressurized without deformation. Another advantage of lunar line is that it does not suffer from the dielectric losses and the bead flashover by insulators as does rigid coaxial line.

The attenuation constants of lunar line and T-septate lunar line are greater than that of the rectangular waveguide, but $\alpha \ll \beta$ and a lost tangent,[5,10] tan $\delta_l \ll 1$, still hold. Therefore, the dispersion due to phase velocity or phase constant will be approximately the same as rectangular waveguide.

The fabrication of lunar line and T-septate lunar line is easier than rectangular ridge waveguide.

From a practical consideration, lunar line and T-septate lunar line are likely to have their greatest applications in 200–2000 Mc frequency band.

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[9] T. N. Anderson, "Rectangular and ridge waveguide," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 207; October, 1956.

[10] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co. Inc., New York, N. Y., pp. 182; 1960.